

AD-A095 272 RENSSELAER POLYTECHNIC INST TROY N Y  
FRACTURE MECHANICS AND DYNAMIC RESPONSE OF STRUCTURES.(U)  
NOV 80 J C AMAZIGO DAAG29-77-G-0198

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FRACTURE MECHANICS AND DYNAMIC  
RESPONSE OF STRUCTURES

FINAL REPORT

J.C. Amazigo

November, 1980

U.S. ARMY RESEARCH OFFICE

GRANT NO. DAAG 29-77-G-0198

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FEB 20 1981  
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STATEMENT OF THE PROBLEMS STUDIED AND SUMMARY OF  
IMPORTANT RESULTS

Much of the work done under this grant is in the area of elastic-plastic crack analysis. Before discussing the results in this area let us note, in passing, the problem in elastic stability which was also supported by the grant. This problem concerns the optional shape of simply supported shallow circular arches against snap buckling. In this work [1] a method which combines perturbation analysis, variational calculus and numerical solution of linear integral equations is developed for the optimization against buckling of structures which undergo limit load buckling. This problem differs from those generally available in the literature by the fact that snap-buckling phenomena cannot be characterized by the extremization of a suitable Rayleigh quotient. The results which indicate that the buckling load of arches may be increased by more than 10 percent by a redistribution of the material of the arch have limited design value; however, the methods developed can be applied successfully to other structures that exhibit snap buckling.

In the area of plastic crack analysis a number of investigations were carried out. Several of these investigations are still going on. We shall report here on the findings and mention some difficulties and unresolved problems.

In one study [2] the near crack tip stress field and J integral are obtained for an infinite strip of material of width b with an edge crack of width a inclined at an arbitrary angle to the edge.

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The material is assumed to have the pure power hardening constitutive relation

$$\gamma/\gamma_0 = \alpha(\tau/\tau_0)^n \quad (1)$$

where  $\alpha$  is a non-dimensional constant,  $\tau_0$  and  $\gamma_0$  are reference values of the principal shear stress and principal strain. The strip of material is subjected to remotely applied out-of-plane shear stress  $\tau_\infty$ . This is an extension of previous analyses [3-5] employing the methods developed in those studies. From these results in [2], simple and accurate formulas may be obtained for the J integral for  $a/b \geq \frac{1}{2}$ .

For values of the hardening parameter  $n$  between  $1/3$  and  $3$ , the quantity  $J_n(\beta)/J_n(\pi/2)$  does not vary much with  $n$ .  $J_n(\beta)$  is the value of J integral corresponding to a crack making an angle  $\beta$  with one of the stress free edges of the infinite strip.

Thus

$$J_n(\beta) \approx a\tau_\infty\gamma_\infty \frac{\pi}{2} \left(\frac{\beta}{1-\beta}\right)^{1-2\beta/\pi} J_n(\pi/2)$$

$$1/3 < n < 3, \quad a/b > \frac{1}{2}.$$

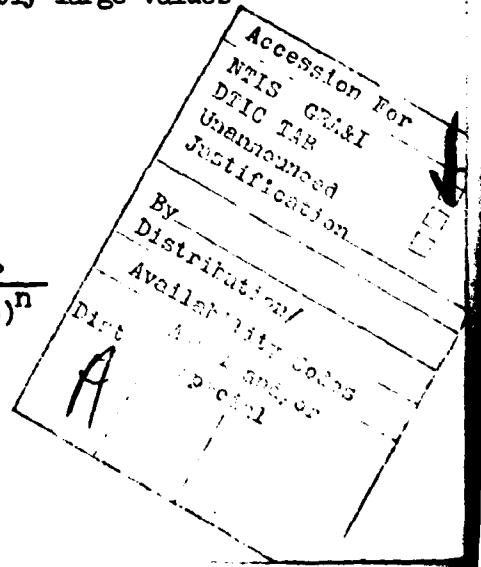
The values of  $J_n(\pi/2)$  are given in Table 1 of reference [5].

In addition, for  $a/b > \frac{1}{2}$  and for sufficiently large values of  $n$  ( $n > 20$ ),

$$J_n(\beta)/J_n(\pi/2) \approx \sin^2\beta,$$

where by [4]

$$J_n(\pi/2) \approx \left(\frac{\pi}{2}\right)^{3/2} \frac{b/a}{\sqrt{n} \exp((n+1)/2n)} \frac{a\tau_\infty\gamma_\infty}{(1-a/b)^n}$$



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thus yielding the following simple formula for  $J_n(\beta)$  for small strain hardening:

$$J_n(\beta) = \frac{a\tau_\infty Y_\infty}{(1-a/b)^n} \left(\frac{\pi}{2}\right)^{3/2} \frac{b/a}{\sqrt{n} \exp((n+1)/2n)} \sin^2 \beta, \quad n \geq 20.$$

Another analysis carried out in the area of fully plastic stationary crack problems was in assessing the range of validity of the dominant term in the singular field solution. In this study [6] we extracted the first two terms in the asymptotic representation of the strain ahead of the crack in a fully plastic center-cracked strip under remote antiplane shear loading and compared it with the exact solution [4]. It is shown, for example, that the dominant term under-estimates the strain ahead of the crack tip by more than 10% for distances one fortieth the half crack width for the power hardening parameter  $n = 5$ . The range of validity is even less for  $n > 5$ . Since this is one of the few classes of plastic crack problems for which exact solutions are obtainable the results raise some concern about the considerable reliance upon the dominant singular field solution for elastic-plastic problems. Further study in this area is essential.

Another area of study is the singular field at the tip of a steadily growing crack. An extension of the pioneering work of McClintock and his coworkers (see for example [7]) had been carried out [8] in collaboration with Hutchinson for materials with linear strain-hardening. This latter work was used [6] in exhibiting the mathematical problems arising in elastic-plastic crack growth problems

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and some of the techniques that have been successfully applied to these areas. Here, the antiplane shear problem is formulated in terms of a stress rate function  $\phi$  whose dominant behavior is sought in the form

$$\phi \sim \tau_0 r^p (\ln r)^\alpha f(\theta) \quad (2)$$

where  $\tau_0$  is the yield shear stress,  $(r, \theta)$  are polar coordinates centered at the crack tip. It is shown that the appropriate forms for  $\phi$  are

$$\phi \sim \begin{cases} K \tau_0 \ln r \cos \theta & \alpha = 0 \\ K \tau_0 r^p f(\theta) & \alpha > 0 \end{cases}$$

where  $\alpha = G_p/G$  is the ratio of the plastic to the elastic shear moduli and  $p$ , as given in [8], approaches 0 like  $\sqrt{\alpha}$  as  $\alpha$  approaches 0.  $K$  is an undetermined intensity factor. This result dramatizes the nonuniformity in the transition of the problem for  $\phi$  from hyperbolic ( $\alpha = 0$ ) to elliptic ( $\alpha > 0$ ), indicating that to obtain a uniformly valid asymptotic result for small strain hardening,  $\alpha \ll 1$ , may require a nontrivial analysis. The author has failed in his attempts to obtain such a uniformly valid asymptotic result.

Two problems that were contained in the original proposal and which occupied the author for a considerable period appear to have been solved by others. The first is the determination of the stress intensity factor  $K$  and its dependence on hardening for the steady crack-growth of a bilinear material under antiplane shear. This has been solved numerically by Dean and Hutchinson [9].

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The other problem is the determination of the singular field at the tip of a steadily growing crack for an elastic-plastic material with a power hardening law in the plastic range. This problem appears to have been solved for plane strain by Yu-Chen and Keh-Chih in an as yet unpublished report [10]. However, in a preliminary study of this latter problem for antiplane loading, Hutchinson and the author are able to obtain the same singular behavior,  $\tau \sim (\ln \frac{1}{r})^{2/(n-1)}$ , for example. However, this does not appear to yield a consistent stress field at the crack tip. Further analysis is being carried out.

LIST OF PUBLICATIONS AND TECHNICAL REPORTS PUBLISHED

1. Amazigo, J.C., "Optional shape of shallow circular arches against snap-buckling", J. Applied Mech., Vol. 45, No. 3, 1978, pp 591-594.
2. Amazigo, J.C., "Asymmetric edge crack in a fully plastic infinite strip", to appear in Proceedings of the First National Colloquium on Mathematical Physics, Dec. 1979
3. Amazigo, J.C., "Some mathematical problems of elastic-plastic crack growth", SIAM-AMS Proceedings on Fracture Mechanics, Vol. XII, 1979, pp 125-136.

## PARTICIPATING SCIENTIFIC PERSONNEL

Professor J.C. Amazigo.

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4. Amazigo, J.C. "Fully plastic center-cracked strip under antiplane shear", Ibid, Vol. 11, 1975, pp. 1291-1299.
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7/14/82. 2-5

## SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER 1104E	2. GOVT ACCESSION NO. AD-A093 272	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Fracture Mechanics and dynamic response of structures.	5. TYPE OF REPORT & PERIOD COVERED 9 Final Rept.	
7. AUTHOR(s) 10) J.C./Amazigo	6. PERFORMING ORG. REPORT NUMBER 15 DAAG29-77-G-0198	
9. PERFORMING ORGANIZATION NAME AND ADDRESS Rensselaer Polytechnic Institute Troy, N.Y. 12181	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 11) 181	
11. CONTROLLING OFFICE NAME AND ADDRESS U.S. Army Research Office Post Office Box 12211 Research Triangle Park, NC 27709	12. REPORT DATE November 1980	
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) Department of the Navy Office of Naval Research Resident Representative, 715 Broadway (5th Floor) New York, N.Y. 10003	13. NUMBER OF PAGES 8	
16. DISTRIBUTION STATEMENT (of this Report)  Approved for public release; distribution unlimited.	15. SECURITY CLASS. (of this report) Unclassified	
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)  NA	16a. DECLASSIFICATION/DOWNGRADING SCHEDULE NA	
18. SUPPLEMENTARY NOTES  The findings in this report are not to be construed as an official Department of the Army position, unless so designated by other authorized documents.		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number)  Elastic-plastic stable crack growth, fully plastic crack fields, J integral, optimization against snap buckling.		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number)  1. A method is developed for the optimization against buckling of structures that undergo snap buckling. 2. Simple formulae are obtained for the dependence of J integral on the angle or edge crack in an infinite strip of fully plastic material under antiplane shear. 3. An assessment is made on the range of validity of the dominant term in the singular field solution for stationary cracks. .../2.		

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4. The mathematical problems arising in the elastic-plastic crack growth problems are highlighted.

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